

Solve the following differential equations.

SCORE: ____ / 30 PTS

[a] $y dy + (y^2 + xy^4) dx = 0$

13 POINTS

FINAL ANSWER:

$$y^{-2} = -x - \frac{1}{2} + Ce^{2x}$$

[b] $2r dr + (1 - r^2 \tan \theta) d\theta = 0$

10 POINTS

FINAL ANSWER:

$$r^2 \cos \theta + \sin \theta = C$$

[c] $\frac{dx}{dt} = \frac{x(x^2 - 3t^2)}{t(x^2 - t^2)}$

8 POINTS

FINAL ANSWER:

$$t^6 e^{\frac{x^2}{t^2}} = Cx^2$$

[a] $\frac{dy}{dx} + y = -xy^3$ BERNOLLI

$$v = y^{-3} = y^{-2}$$

$$\frac{dv}{dx} = -2y^{-3} \frac{dy}{dx}$$

$$-2y^{-3} \frac{dy}{dx} - 2y^{-2} = 2x$$

$$\frac{dv}{dx} - 2v = 2x$$

$$\mu = e^{\int -2 dx} = e^{-2x}$$

$$e^{-2x} \frac{dv}{dx} - 2e^{-2x} v = 2xe^{-2x}$$

CHECK: $\frac{d}{dx} e^{-2x} = -2e^{-2x}$ ✓

$$e^{-2x} v = \int 2xe^{-2x} dx + C$$

$$= -xe^{-2x} - \frac{1}{2}e^{-2x} + C$$

$$v = -x - \frac{1}{2} + Ce^{2x}$$

$$y^{-2} = -x - \frac{1}{2} + Ce^{2x}$$

$$\begin{array}{r} \frac{v}{2x} + \frac{dv}{e^{-2x}} \\ 2 \quad - \frac{1}{2}e^{-2x} \\ 0 \quad \frac{1}{4}e^{-2x} \end{array}$$

NO PARTIAL CREDIT IF YOUR CHECKPOINT WAS MISSING, INCORRECT OR ROGUS

$$[b] \quad M = 2r \quad N = 1 - r^2 \tan \theta$$

$$M_\theta = 0 \quad N_r = -2r \tan \theta$$

$$\frac{N_r - M_\theta}{M} = \frac{-2r \tan \theta}{2r} = -\tan \theta \quad \text{FUNCTION OF ONLY } \theta$$

$$\mu = e^{\int -\tan \theta d\theta} = e^{\ln |\cos \theta|} = \cos \theta$$

$\uparrow$
 $u = \cos \theta$
 $du = -\sin \theta d\theta$

$$\underbrace{2r \cos \theta dr}_M + \underbrace{(\cos \theta - r^2 \sin \theta) d\theta}_N = 0$$

$$M_\theta = -2r \sin \theta = N_r \quad \text{EXACT } \checkmark$$

$$f = \int 2r \cos \theta dr = r^2 \cos \theta + C(\theta)$$

$$f_\theta = -r^2 \sin \theta + C'(\theta) = \cos \theta - r^2 \sin \theta$$

$$C(\theta) = \sin \theta$$

$$r^2 \cos \theta + \sin \theta = C$$

$$[c] \quad \underbrace{t(x^2 - t^2)}_M dx + \underbrace{x(3t^2 - x^2)}_N dt = 0$$

$$M(kx, kt) = kt(k^2x^2 - k^2t^2) = k^3t(x^2 - t^2) = t^3 M(x, t)$$

$$N(kx, kt) = kx(3k^2t^2 - k^2x^2) = k^3x(3t^2 - x^2) = t^3 N(x, t)$$

NO PARTIAL CREDIT
IF YOUR CHECKPOINT
WAS MISSING,
INCORRECT
OR BOGUS
HOMOGENEOUS

$$x = vt \quad dx = v dt + t dv$$

$$t(v^2t^2 - t^2)(v dt + t dv) + vt(3t^2 - v^2t^2) dt = 0$$

$$(v^3t^3 - vt^3 + 3vt^3 - v^3t^3) dt + (v^2t^4 - t^4) dv = 0$$

$$2vt^3 dt + (v^2 - 1)t^4 dv = 0$$

$$\int \frac{1}{t} dt = \int \frac{1-v^2}{2v} dv = \int \left(\frac{1}{2} \frac{1}{v} - \frac{1}{2} v \right) dv$$

$$\ln |t| = \frac{1}{2} \ln |v| - \frac{1}{4} v^2 + C$$

$$4 \ln |t| = 2 \ln \left| \frac{x}{t} \right| - \left(\frac{x}{t} \right)^2 + C$$

$$t^4 = \frac{Cx^2}{t^2} e^{-\frac{x^2}{t^2}}$$

$$t^6 e^{\frac{x^2}{t^2}} = Cx^2$$

OR

ALTERNATE SOLUTION FOR [b]

★ USE ONLY 1 VERSION FOR GRADING

$$\frac{dr}{d\theta} - \frac{1}{2} r \tan \theta = -\frac{1}{2} r^{-1} \quad \text{BERNOULLI}$$

$$v = r^{1-1} = r^2$$

$$\frac{dv}{d\theta} = 2r \frac{dr}{d\theta}$$

$$2r \frac{dr}{d\theta} - r^2 \tan \theta = -1$$

$$\frac{dv}{d\theta} - v \tan \theta = -1$$

$$\cos \theta \frac{dv}{d\theta} - v \sin \theta = -\cos \theta$$

$$(\cos \theta) v = \int -\cos \theta d\theta + C$$

$$= -\sin \theta + C$$

$$v = -\tan \theta + C \sec \theta$$

$$r^2 = -\tan \theta + C \sec \theta$$

OR

$$r = v^{\frac{1}{2}} \rightarrow \frac{dr}{d\theta} = \frac{1}{2} v^{-\frac{1}{2}} \frac{dv}{d\theta}$$

$$\frac{1}{2} v^{-\frac{1}{2}} \frac{dv}{d\theta} - \frac{1}{2} v^{\frac{1}{2}} \tan \theta = -\frac{1}{2} v^{-\frac{1}{2}}$$

$$\mu = e^{\int -\tan \theta d\theta} = e^{\ln |\cos \theta|} = \cos \theta$$

$$\text{CHECK: } \frac{d}{d\theta} \cos \theta = -\sin \theta \quad \checkmark$$

NO PARTIAL CREDIT

IF YOUR CHECKPOINT

WAS MISSING, INCORRECT

OR BOGUS

OR

ALTERNATE SOLUTION FOR [c]

★ USE ONLY 1 VERSION FOR GRADING

$$\underline{t = vx} \quad \underline{dt = v dx + x dv}$$

$$\underline{vx(x^2 - v^2x^2)dx + x(3v^2x^2 - x^2)(v dx + x dv) = 0}$$

$$(\cancel{vx^3} - v^3x^3 + 3v^3x^3 - \cancel{vx^3})dx + (3v^2x^4 - x^4)dv = 0$$

$$\underline{2v^3x^3 dx + x^4(3v^2 - 1)dv = 0}$$

$$\underline{\int \frac{1}{x} dx = \int \frac{1 - 3v^2}{2v^3} dv = \int (\frac{1}{2}v^{-3} - \frac{3}{2}v^{-1}) dv}$$

$$\underline{\ln|x| = -\frac{1}{4}v^{-2} - \frac{3}{2}\ln|v| + C}$$

$$\underline{4\ln|x| = -\left(\frac{t}{x}\right)^2 - 6\ln\left|\frac{t}{x}\right| + C}$$

$$x^4 = \frac{Cx^6}{t^6} e^{-\frac{x^2}{t^2}}$$

$$\underline{t^6 e^{\frac{x^2}{t^2}} = Cx^2}$$

OR $(tx^2 - t^3)dx + (3t^2x - x^3)dt = 0$ ^{2ND} ALTERNATE SOLUTION FOR [c]

GUESS $\mu = t^a x^b$

★ USE ONLY 1 VERSION FOR GRADING

$$\underbrace{(t^{a+1}x^{b+2} - t^{a+3}x^b)}_M dx + \underbrace{(3t^{a+2}x^{b+1} - t^a x^{b+3})}_N dt = 0$$

$$\begin{aligned} M_t &= (a+1)t^a x^{b+2} - (a+3)t^{a+2} x^b \\ N_x &= 3(b+1)t^{a+1} x^b - (b+3)t^a x^{b+2} \end{aligned}$$

$$a+1 = -(b+3)$$

$$-(a+3) = 3(b+1)$$

$$-2 = 2b \rightarrow b = -1$$

$$a = -3$$

$$\mu = t^{-3} x^{-1}$$

$$\underbrace{(t^{-2}x - x^{-1})}_M dx + \underbrace{(3t^{-1} - t^{-3}x^2)}_N dt = 0$$

$$M_t = -2t^{-3}x = N_x \quad \text{EXACT } \checkmark$$

$$f = \int (t^{-2}x - x^{-1}) dx$$

$$= \frac{1}{2}t^{-2}x^2 - \ln|x| + C(t)$$

$$f_t = -\cancel{t^{-3}x^2} + C'(t) = 3t^{-1} - \cancel{t^{-3}x^2}$$

$$C(t) = 3\ln|t|$$

$$\frac{1}{2}t^{-2}x^2 - \ln|x| + 3\ln|t| = C$$

$$\frac{t^3}{x} e^{\frac{x^2}{2t^2}} = C$$

$$t^3 e^{\frac{x^2}{2t^2}} = Cx$$

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